Informational Industrial Blackmail

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Abstract

We develop a model to explain the provision of public subsidies for private investment by firms. The model relies on the concept of 'signal-jamming' by incumbent politicians whose re-election prospects are improved by the offering of such subsidies. The model implies that such subsidies arise in equilibrium only under some conditions, an important feature, given that not all private investment in the real world receives such subsidies.

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1 Introduction

That governments at all levels regularly provide public funds for, or otherwise subsidize, private investment projects is not news. One often-analyzed instance of this is the subsidization of the construction and sometimes operation of facilities used by professional sports teams, and the amounts involved are not small. Long (2005) estimated that the MLB ballparks in use in 2001 received a mean of $218 million each (in 2001 dollars) in public subsidies that include purchasing land, infrastructure, and providing ongoing operating subsidies.[1] A Cato Institute Analysis from 1999 (Keating, 1999) estimated that some $20billion (in 1997 dollars) had been spent on professional sports facilities in North America up to that point, with $14.7 billion - almost 3/4 - coming from government funds, and also pointed out that there were already plans to spend another $13.5billion, with $9 billion of that coming from taxpayers in some form.

Professional sports teams are not the only firms to receive such subsidies, and are far from being the largest recipients. Good Jobs First, an organization that tracks government subsidies to private firms, issued a 'Mega Deals Report' (2013) which lists 11 projects that received more than $1billion dollars each in subsidies; none of them went to sports facilities, as automotive and energy companies were most prominent on the list. The same report notes that in terms of the number of 'mega-deals'[2] to receive subsidies, GM led the list with 11, followed by Ford with 9.

Jensen, Malesky and Walsh (2015) cite a survey conducted by the International City/County Management Association (ICMA) in 2009 which found that 95% of US municipalities offered targeted incentives of some kind to firms. The Council for Community and Economic Research (C2ER) un-

[1] These are calculated as the present discounted value of all government subsidies, grants and tax holidays.

[2] Defined as projects that receive $75million or more in government subsidies of any sort.

dertook a nationwide survey of state incentive program managers in 2012 and found that the number of state-level economic incentive *programs* had increased from 940 in 1999 to 1,799 in 2012. This suggests that state and local governments find the offering of such incentives to profit-making organizations to be worthwhile, and seemingly increasingly so.

While there is no database of unsubsidized firm investments, it is also clear that not every firm that locates in or undertakes new investment in a given jurisdiction receives a publicly funded incentive to do so. Even in the construction of facilities for professional sports, where public subsidies and direct spending have become the rule, a 2001 report from the St. Louis Fed (Zaretsky, 2001) notes that 14 of the approximately 140 such facilities built in the US between World War II and 2001 did not receive government funds or subsidies. This paper is an attempt to understand what drives these differences; put differently, what determines which firm investments in a given political jurisdiction receive government subsidies?[3]

An additional feature of this widespread practice is the wide range of views regarding their net benefits, or, to put it differently, the considerable disagreement about who actually benefits from such subsidies. The general view among economists of subsidies for professional sports facilities is rather overwhelmingly negative. An oft-cited (if admittedly small) survey of economists in 2006 (Whaples, 2006) found that 85.2% of respondents believed that subsidies to professional sports teams should be eliminated.[4] A less negative take on the value of industrial incentives in general can be found in a 2008 paper from the Federal Reserve Board of Governors (Gorin, 2008),

[3]We will consistently use the term 'subsidy' to refer to any incentive offered to a firm that is funded by the taxpayer. These incentives can and do take many forms, including tax abatements or reductions, loans at below-market rates, direct public spending and the institution of new taxes (styled as 'community development levies') whose proceeds are used to fund a subsidy.

[4]Seigfreid and Zimbalist (2000) provide a comprehensive analysis of the many ways that professional sports teams and their facilities are claimed to benefit those living in the jurisdictions in which they operate, and conclude that none of them add up to a justification for government subsidies for facility construction.

which concluded that "...studies suggest that incentives can be effective in certain situations,..." but the focus in that work is on a given subsidy's effectiveness at influencing firm investment decisions, rather than in making taxpayers better off. The most prominent negative view of these subsidies arises from a part of the literature that gives rise to the title of our paper. This literature focuses on the idea that subsidies are offered to firms by governments as a means of 'bidding for footloose firms'. Much of the early research on the issue, including the seminal paper by Doyle and van Wijnbergen (1984), utilizes a bidding model that indeed seems an apt description of the current competition among cities to host Amazon's HQ2, as well as the BMW and Mercedes auto plant deals mentioned above. King, McAfee and Welling (1993), among others, have constructed models based on this idea, and they report that it was the mayor of Flat Rock, Michigan who coined the term "industrial blackmail" for the multi-city bidding process in which Flat Rock won a Mazda plant after offering a $120 million subsidy package. Our approach encompasses this view of the phenomenon, but the firm's alternative to investing locally need not be a subsidy offer from another jurisdiction; it may just be the choice to not invest at all, or to continue to operate an existing facility, but in any case only the firm knows how profitable is that alternative.

Our contribution to understanding this phenomenon is primarily positive; we are interested in when such tax-financed incentives are offered, and our model will suggest that whether the expected net benefit to taxpayers from a private project is positive or negative is not a critical factor in triggering the offer of a subsidy. However, consistent with our title, it will emerge from our model that taxpayers can find themselves subsidizing projects that would have gone forward in the absence of the subsidy. This arises due to informational asymmetries, and gives rise to our title. We should note that we are concerned only with *targeted* incentives. Our results do not apply to ongoing government programs that offer a list of particular incentives to all firms/projects meeting some pre-specified criteria (such as newness or size). We seek to understand situations in which a government policy-maker offers a subsidy to a particular firm that is tailored to that firm undertaking a particular investment within its jurisdiction. This encompasses a great many instances of such incentives, including the ones that generate the biggest headlines. All subsidies to professional sport facilities are encompassed by our model, as was BMW's 1992 $150 million package of incentives to locate a plant in South Carolina, and Mercedes-Benz's reported $258 million package the next year to locate a production facility in Alabama.[5] Amazon's drawn out 'shopping around' for a location for their HQ2 is another notable example of the phenomenon we study.

The starting point for our model is to be found in empirical work on these subsidies by Jensen, et al. (2014), Jensen, et al. (2015), and Jensen and Malesky (2018), which examines how electoral motivations encourage the use of industrial incentives by politicians. In particular Jensen, et al. (2015) evaluate the incentive programs of US cities under different electoral regimes – namely, cities with directly elected mayors versus cities with non-elected city managers. Exploiting exogenous random assignment of political institutions, they find that cities with elected mayors offer incentives with greater frequency, and offer more generous incentive packages than do their non-elected counterparts. Moreover, the use of these incentives increases significantly in election years. Thus, a critical component of our model is that the policy-maker (who we will refer to as a 'mayor') wishes to be re-elected to office, and does not personally pay for any subsidy. The citizenry, on the other hand, only wants to re-elect the incumbent if they believe he will be better for them than a challenger, and they do have to finance any subsidy the mayor offers. The other aspect of our model that is based in the empirical

[5]Good Jobs First is a nonprofit that keeps track of public subsidies to firms. A study of the location of automobile assembly plants can be found on their website at: www.goodjobsfirst.org/corporate_subsidy/automobile_assembly_plants.cfm.

findings in the literature is that we model the firm's investment project as being *productive* from the perspective of the taxpayer. In our model, the initiation and operation of the project has a stochastic but positive impact on the well-being of the citizenry; the literature on these projects does not dispute the existence of benefits to (some) taxpayers from them, but on whether those benefits are greater than the subsidy's cost.

We are therefore putting forward a political economy explanation for the existence of such subsidies, but one that turns specifically on the electoral incentives of policy-makers. This differs fundamentally from an older explanation due to Mancur Olson (Olson, 1965), which relies on the observation that most such projects feature gross benefits that are concentrated among a few recipients, who are thus willing to spend considerable resources lobbying for a subsidy, while the costs of any subsidy are spread thinly over many taxpayers. Whatever the merits of this explanation, it has the empirical implication that almost every project will be subsidized, whereas our model makes a set of more nuanced predictions which we will show better conform to the findings in most existing empirical work on such incentives.

We model a strategic interaction between a government decision-maker (the 'mayor'), assumed to be purely office-motivated, a firm which might invest in some project located in the mayor's city, and a representative voter who will finance any subsidy, and possibly benefit from the investment, but who also determines whether the mayor will remain in office. In our model the key driver of the mayor's decision to offer a subsidy to the firm, as well as the firm's decision to accept it and make a costly (sunk) investment, is informational. The voter is uncertain about the ability of the incumbent mayor, and that ability has a stochastic impact on the voter's ongoing well-being. The project, if it is undertaken, also has a positive but stochastic impact on the voter's well-being, and in equilibrium the mayor offers the subsidy in order to favorably influence the voter's beliefs about his ability, and thereby increase the probability of re-election. The firm's beliefs about the mayor's re-election prospects also matter, since the future defeat of the mayor in an election has a negative impact on the promised subsidy. Additionally, although all players know the profitability of the local project for the firm, only the firm knows how that compares to the profitability of whatever would be the firm's next best alternative if the local project is not undertaken. The key to our model's prediction is that idea that subsidizing a private project allows a mayor to engage in 'signal-jamming', since it increases the likelihood the voter is well off during the mayor's term in office, and the voter cannot completely untangle the extent to which her good fortune is due to having a competent mayor or a stochastically beneificial project.

In the context of a professional sports team, the local project could be the construction of a new facility in which to operate; only the team's owners know how the profitability of the proposed new local arena compares to either continuing to operate in an existing facility or to moving the team to a new city (where it might have already been offered a particular incentive package). The mayor must decide whether to subsidize the construction of the new facility using tax dollars, knowing that the citizens will observe the offer if he makes it. The citizens, unsure of the quality of the mayor, will update their beliefs based on whether a subsidy is offered, how the firm reacted to it and how well they themselves have done during the incumbent mayor's tenure so far. Citizens then decide whether to re-elect the mayor, knowing that they are on the hook for the cost of the subsidy if indeed the mayor offered one, the firm accepted it, and he is re-elected, but also knowing that they can reduce their tax burden by electing a challenger of unknown quality, who will (perhaps partly) renege on the subsidy. Thus, their updated pre-election beliefs about their mayor, as well as their beliefs about what the firm will do if the subsidy is reduced, are key to their electoral decision, and those in turn depend on the actions of the mayor and firm and on the informative signal they received regarding the incumbent mayor's quality before the election.

The next section places our paper within the literature, and then Section III presents our formal model and results. Section IV discusses its implications for the world of public subsidies of private projects, and the last section concludes.

2 Literature

Our paper fits, in a very general sense, into the vast theoretical literature on tax competition among jurisdictions, although, as noted above, our model is one of specific tax (or other) incentives directed at specific investments by specific firms. Most of the tax competition literature takes overall tax rates on capital or on firms to be the jurisdiction's strategic variable. In addition, these models of tax competition either take the policy-maker to be motivated to maximize a utilitarian social welfare function, or assume the tax rates are set by a median voter. The politics in our model is instead driven by 'political agency', in the tradition of models that date back to Austen-Smith and Banks (1989) and Rogoff (1990), with the policy-maker and representative voter being separate strategic players, and does not require that there be more than one jurisdiction competing for the firm's investment.

The literature on *targeted* incentives began with the seminal Doyle and van Wijnbergen(1984) paper, and includes contributions by Bond and Samuelson (1986), Black and Hoyt (1989), King and Welling (1992), and King, McAfee and Welling (1993), all of which adopt some variant of a model of jurisdictions entering an auction, in which the prize is an investment in their jurisdiction by some 'footloose' firm. In none of these papers is there any discussion of the possibility that the winning 'bid' is zero, hence the emphasis is not on when such subsidies are offered, but rather on their size or structure when they are.

The paper that is closest to ours is that of Biglaiser and Mezzetti (1997), in which a politician of unknown ability decides whether or not to fund a project which has a stochastic impact on voter well-being. As in our model, the politician's ability also has such an impact, and voters decide whether to re-elect the politician after observing what happens after the politician's project-funding decision. They derive the politician's 'willingness to pay' for a project and their main result is to show that it differs, in general, from the voter's willingness to finance it. Voters who have an inherent bias against the incumbent will be willing to pay less for the project than the incumbent is, and the opposite is true if voters have an inherent bias in favor of the incumbent. There is no firm in their model, so that the project is any publicly funded project, and the only source of information for the voter about the politician's ability is the first period outcome of the project, but *only if* the politician funds it. Thus, the politician is in effect determining how much she will pay to have that information generated, leading to the prediction that an incumbent politician will fund the project only if she is in electoral difficulty. Our model makes a different prediction: it is electoral competition itself that generates a willingness to subsidize a private firm's local project.

Jensen and Malesky (2018) do not develop a formal model to structure the data and empirical findings in their book, but they indicate that they think it is best explained by a model of 'pandering' by politicians. To quote them, "We argue that in order for politicians to pander, voters need not hold incorrect views about incentives; they need only to be uncertain enough about the policies' ultimate benefits so that a politician can persuade them in a campaign." So a key ingredient in their thinking about this phenomenon is that voters need to be uncertain about the actual benefits of the investment project being subsidized; our model assumes that those benefits are indeed uncertain, but all players have accurate expectations about them, and the same is true about the (private) ability of the mayor and the (private) profitability of the firm's alternative project. Thus, in our model, no one is being fooled, and the incentives, if they arise, are the result of asymmetric information about the policy-maker and firm.

3 Model

We assume a two-period time frame without discounting, with a 'mayor' in office in a single jurisdiction in period $t = 1$, who must stand for re-election at the end of that period. In each period there is a state of nature that is realized, which we denote as $\omega_t \in \{0, 1\}$ for $t = 1, 2$. It will be apparent below that state 1 is better for the city's representative voter.

We refer to the mayor as player M, whose ability, θ, is either $l$ow or $h$igh, so $\theta \in \{l, h\}$ and the mayor of type θ is referred to as M_θ. We assume that it is common knowledge that M is equally likely to be of either type. The mayor is purely office-motivated, earning a payoff of 1 if re-elected for a second term and 0 if not. Being risk-neutral, M_θ's payoff function is thus p^e, the probability of re-election. The one decision to be made by M_θ is whether or not to offer an action-contingent subsidy to the firm.

The single voter's payoff is $\omega_1 + \omega_2 - T$, where T is any incremental tax payment created by decisions made by M. The voter has one decision to make at the end of period 1, denoted by $e \in \{0, 1\}$ with 1 being a decision to re-elect M and 0 a decision to replace him with a challenger. The challenger's ability can also be either l or h, and p_h is the exogenously given probability the challenger's type is h. We make no assumption about how p_h compares to 1/2; we will put more structure on the uncertainties in the model below.

The third strategic agent is the firm of type τ, denoted as F_τ, where $\tau \in \{L, H\}$; the meaning of the firm's type is detailed below.

The firm makes an initial choice $d \in \{\phi, I\}$, which is a decision to make an initial investment in the city (I) or not (ϕ). Choosing $d = I$ incurs a sunk cost γ in period 1; these are any unrecoverable outlays by the firm made prior to the operation of the project in period 2. In period 2, if the Firm incurred γ, it then decides whether to continue the project ($x = 1$) or not

$(x = 0)$. If $d = \phi$ we assume $x = 0$ necessarily; there is no project to continue. Choosing $x = 1$ implies the Firm incurs a further period 2 one-time cost of η and generates ongoing operating profits from the project with a present value discounted to period 2 of π_I. If the Firm chooses $x = 0$, then it doesn't pay η in period 2 and generates ongoing operating profits with a period 2 discounted present value of π_τ. It is this, the profitability of the outside option, that depends on the Firm's type, which is private information to the firm.

We assume in particular, that

$$\pi_H > \pi_I - \gamma - \eta > \pi_L$$

Thus, the L-type firm is distinguished by the fact that the best alternative it has to undertaking the local project is inferior to it. Firm F_L thus has every incentive to make the new local investment in the city. The H type firm on the other hand has an alternative to the local investment that is more profitable. We assume that $q = \Pr\{\tau = H\} \in [0, 1]$ is common knowledge.

We will in fact make the stronger assumption that $\pi_H > \pi_I - \eta$: any extra operating profits generated by the local project do not cover even its continuation costs, which of course implies the H type Firm does not wish to incur the initial γ, either.[6] In the above example of the sports team considering building a new arena in the city, π_τ may be the future profit stream generated by moving the team to another city, but it may also be generated by continuing to operate the existing local arena. The key is that only the Firm knows which it is, and the value of π_τ it generates.

The final element of the model is the subsidy, s, which the Mayor may offer the Firm. We do not attempt to model a possible bargaining process between the Firm and Mayor regarding the size of a subsidy, and limit the

[6]Note that it can still be that the operating profits from operating the project from period 2 on are superior to not doing so, so that $\pi_I > \pi_H$. The hockey team may in fact earn greater operating profits after the land for a larger arena is developed and rezoned (γ) and the arena is constructed (η).

Mayor to a stark choice. He can offer no subsidy at all, so $s = 0$, or offer a subsidy $v > 0$ that has the property that $\pi_I - \gamma - \eta + v > \pi_H$. That is to say, the Mayor either offers a subsidy large enough to get even F_H to invest locally in the new project, or offers nothing. Of course this means that firm F_L, if offered this same subsidy, will be more than happy to accept it, also. Since the cost of the subsidy is covered by the voter-as-taxpayer, the Voter's payoff is in fact $\omega_1 + \omega_2 - s$.

This simplification means that we can model the strategy of the Mayor as $\sigma = (s_l, s_h) \in \{0, v\}^2$.

Thus our full assumptions about firm types and the subsidy are as below:

$$\pi_I - \gamma - \eta + v > \pi_H > \pi_I - \gamma - \eta > \pi_L \tag{1}$$

While neither the voter nor the firm can observe θ, the voter does see ω_1, s and d before choosing e, her re-election decision. This matters because it is common knowledge that both θ and d have an impact on the realization of ω_1.

Specifically, we assume the probability that $\omega_1 = 1$ for any given θ, d pair is denoted by $p_{\theta d} \in (0, 1)$, and has these properties:

$$\begin{aligned} p_{hd} &> p_{ld}, \text{ for } d \in \{\phi, I\}\text{, and} \\ p_{\theta I} &> p_{\theta\phi}, \text{ for } \theta \in \{l, h\}. \end{aligned} \tag{2}$$

Thus, a high ability incumbent has a positive impact on expected voter well-being in period 1, whether or not the Firm initiates the local investment. Additionally, that initial investment of γ in the local project by the firm also has a positive impact on expected voter payoff in period 1(as will the continuation of the project on period 2 payoffs, as we detail below). In this sense, this local investment by the Firm is *productive*. Note, however, that this is not the same as assuming the voter is happy to subsidize the project, an issue we address below.

The voter does have a say in the subsidization of the firm's project even if M has offered $s = v$ and the firm has chosen $d = I$ and made its sunk investment. The voter can vote a challenger into office in the second term. If this happens, we assume that the payment of the subsidy to the firm is cancelled. This implies that the H type firm chooses not to continue the project in period 2, since $\pi_H > \pi_I - \eta$.[7] A type L firm that chose $d = I$ at period 1 however, will choose $x = 1$ even if the subsidy is pulled back or reduced, since the local project was its best alternative without any subsidy.

We assume that the impact of the firm choosing $x = 1$ on the realization of ω_2 is the same as the effect of choosing in period 1, so that, letting $p_{\theta x}$ be the probability that $\omega_2 = 1$ conditional on θ and, now, x, we assume that

$$p_{\theta 1} = p_{\theta I}, \text{ and } p_{\theta 0} = p_{\theta \phi}.$$

Again, it is not necessary for these probabilities to be identical, what matters is that they again satisfy (**??**); this assumption simply reduces the amount of notation we have to carry around.

As to the impact of the mayor on ω_2, if the incumbent is not re-elected, then his type, θ, has no impact on the probability that $\omega_2 = 1$. Placing a challenger in office whose probability of being high ability is p_h, implies that the probability that $\omega_2 = 1$ becomes

$$p_C^{\phi} \equiv p_h p_{h\phi} + (1 - p_h)\, p_{l\phi},$$

if the project doesn't go forward ($x = 0$), and it is

$$p_C^{I} \equiv p_h p_{hI} + (1 - p_h)\, p_{lI},$$

[7]The assumption that the subsidy is cancelled in the event the challenger is elected is stronger than necessary. It is sufficient for our purposes that election of the challenger implies the subsidy is reduced, to v' with $\pi_H > \pi_I - \eta + v'$, where v' is the expected value of a stochastically reduced subsidy.

if it does.

Our assumptions that initiation and operation of the local project are stochastically good for the voter do not imply that she will be happy to subsidize it.

Suppose one polled the single voter at the start of period 1, asking her: 'Given what you know about the firm, the local project, and your Mayor, are you in favor of your Mayor offering the firm a subsidy of v that is contingent on its carrying out that project?'

Within our model, the voter would answer Yes if and only if

$$\frac{1}{2}\left[2p_{hI}+2p_{lI}\right]-v>q\left\{\frac{1}{2}\left[2p_{h\phi}+2p_{l\phi}\right]+\right\}+(1-q)\left\{\frac{1}{2}\left[2p_{hI}+2p_{lI}\right]\right\} \quad (3)$$

where the LHS is the voter's ex-ante expected two-period payoff with the subsidy offered (and the Mayor re-elected), based on her prior beliefs about the Mayor and Firm, and the RHS is the same, but without the subsidy, so that the project goes forward only if $\tau = L$.

We make no assumption about the sign of this inequality, but will return to it later.

The sequence of moves in the game is as follows:

1. Nature chooses M's type, $\theta \in \{l, h\}$, which only M observes, and F's type $\tau \in \{L, H\}$, which only F observes.
2. M_θ chooses $s_\theta \in \{0, v\}$.
3. F_τ observes the value of s, and chooses $d_\tau(s) \in \{\phi, I\}$.
4. The state $\omega_1 \in \{0, 1\}$ is realized, with $p_{\theta d} = Pr\{\omega_1 = 1\}$.
5. The voter (V) observes s, d and ω_1 and chooses $e \in \{0, 1\}$.
6. F_τ observes d, s, e and chooses $x \in \{0, 1\}$ (with $x = 0$ necessarily, if $d = \phi$)

7. The state ω_2 is realized according to the following probabilities:

   (i) If V chose $e = 1$, then $\omega_2 = 1$ with probability $p_{\theta x}$,

   (ii) If V chose $e = 0$, then $\omega_2 = 1$ with probability p_C^d.

8. Player payoffs are realized.

The payoffs to each player in this game, expressed so as to explicitly recognize their dependence on all players' strategies, are:

$$\begin{aligned} M_\theta &: \; U_\theta = e \\ F_\tau &: \; U_{F\tau} = \varphi(d)\left[x\left(\pi_I - \eta + se\right) - \gamma\right] + (1-x)\,\pi_\tau \\ V &: \; U_V = \omega_1 + \omega_2 - esx\varphi(d) \end{aligned}$$

where

$$\varphi(I) = 1 - \varphi(\phi) = 1$$

and all players are assumed risk-neutral.

Note that the firm receives the subsidy v (and the voter finances it) only if all of $s = v$, $d = I$ and $e = x = 1$ are chosen.

We will determine the set of perfect Bayesian equilibria (henceforth PBE or just 'equilibria') of the model. The derivation will be made simpler by noting the following about stage 6 in the game above. The firm is only in a position to choose $x = 1$ at this stage if it chose $d = I$ at stage 3. If that is the case, the choice of each type of firm is deterministic. For firm H, $x = 1$ if and only if all of $d = I, s = v$ and $e = 1$ hold, and we will simply insert this behavior by F_H into stage 6 of the game above. Similarly, Firm L always chooses $x = 1$ if $d = I$, which we also insert into the game above.

4 Results

Before presenting the main results, we lay out further definitions, and the key assumptions that will be important in solving for the equilibria – in particular regarding voter beliefs and updating about θ; our assumptions will in particular impose structure on the voter's *interim* beliefs about her Mayor and the Firm, after she has observed what happened in period 1.

As to beliefs about the Mayor's type, given the Mayoral strategy σ and after observing s, d and ω_1 in period 1, we write the voter's posterior belief that $\theta = h$ as $\alpha_d(\omega_1, \sigma, s)$. In addition, the Firm will have beliefs about M's type which matter to F because they may influence the likelihood M is re-elected, which in turn determines whether F will see a promised subsidy reduced. These will depend only on (σ, s) and we write $\delta(\sigma, s)$ as the probability F attaches to $\theta = h$ after seeing s, when $\sigma = (s_l, s_h)$.

The voter uses Bayes' rule to determine the probability the incumbent is of high ability ($\theta = h$), after observing s, d and ω_1. If M's strategy reveals nothing about his type, then conditional on the observed d and ω_1,. These posterior beliefs about θ given any non-informative σ will be denoted as $\beta_d(\omega_1) \equiv \alpha_d(\omega_1, (s', s'), s')$ and are defined by:

$$\begin{aligned} \beta_d(0) &= \frac{1 - p_{hd}}{(1 - p_{hd}) + (1 - p_{ld})}, \text{ and} \\ \beta_d(1) &= \frac{p_{hd}}{p_{hd} + p_{ld}} \end{aligned}$$

These beliefs about θ when M's strategy conveys no information allow us to state two key conditions regarding the influence of θ on the state, ω_1, via its effect on the voter's posterior beliefs.

C1: $\beta_d(0) < p_h < \beta_d(1)$ for $d \in \{\phi, I\}$.

This is an assumption that – when the Mayor's behavior is uninformative – the realization of state 1 contains enough information to make a nontrivial difference to the voter's beliefs about the incumbent in relation to an unknown challenger.

The second assumption is similar, but for an environment in which the Mayor has granted a subsidy and the Firm has invested, but again this behavior has revealed no new information about θ.

C2: $\beta_I(0)p_{hI} + [1 - \beta_I(0)]\, p_{lI} < p_C^{\phi} + v < \beta_I(1)p_{hI} + [1 - \beta_I(1)]\, p_{lI}$

In summary, what these two conditions require is that, in the absence of further information about the quality of their Mayor, the voter's re-election decision is responsive to the realization of the state in period 1; how well the voter did during the mayor's term contains enough information about the mayor to have an impact on her vote in the election. These conditions also imply that the election is expected to be *competitive* in that the expected challenger quality is not so low or high to render the voter's observations of the period 1 state irrelevant.

Our final condition relates to the type H firm:

C3: $\left[\frac{1}{2}(p_{hI} + p_{lI})\right] [(\pi_I - \eta + v) - \pi_H] > \gamma$

The first bracketed term on the LHS is the probability that $\omega_1 = 1$ if the firm chooses to invest, given its prior about the Mayor. The second bracketed term is the difference in the Firm's profits from period 2 onward between going forward with the subsidized project and not; by our assumptions above, this difference is positive. Thus, this condition requires these two values to be sufficiently high relative to the sunk cost of the project. It in effect assumes that the H Firm does not need certainty about the subsidy being maintained in period 2 to initiate the local project – it can live with some uncertainty.

These assumptions define an environment in which our model delivers particular equilibrium decisions and outcomes by all parties. It is these that will allow us to generate predictions in the last section of the paper, but some basic things are clear already. In particular, C1 and C2 can be read as saying the Mayor's ability and the project have a *significant enough* influence on the voter's well-being that the signal she gets about them, although imperfect, has an impact on her voting decision.

4.1 Equilibrium when $q = 1$

We begin with a special case of the model since it provides clear intuition on the behavior of the agents in our model, and because it is of interest in its own right. We first lay out the unique PBE under the assumption that $q = 1$, which means that it is common knowledge that the Firm is of type H, and so does not find the local project as profitable as its best alternative. The local sports team will move to another city or continue to operate its existing arena in the absence of a subsidy, or the local manufacturing firm would not find the construction of a new warehouse and distribution centre profitable. This means that the Firm has no interest in the local project and the Voter, who recognizes this fact, also recognizes that if it does go forward it will have a stochastically positive effect on her well-being. One might expect that if in fact condition (**??**) (with $q = 1$) is reversed, then in any PBE the voter will turf out the Mayor if he offers the subsidy, and so in equilibrium the Mayor will not do so. Proposition 1 says that our model delivers a different prediction.

Proposition 1. *If $q = 1$, and conditions (**??**), (**??**) hold, along with conditions C1, C2 and C3, then the only PBE of the game above in pure strategies is the following:*

1. *$\sigma^* = (v, v)$; that is, both types of incumbent mayor offer the subsidy, so $s_l = s_h = v$.*

2. *$d^*_H(v) = I$ and $d^*_H(0) = \phi$; that is, the Firm incurs the sunk project cost, γ, if and only if the subsidy is offered.*

3. *$e^*(\omega_1, s, d) \equiv \omega_1$; that is, the voter re-elects the incumbent if and only if state 1 is realized in the first period, independently of the s, d the voter observes.*

Beliefs along the equilibrium path are given by Bayes Rule. Off the equilibrium path we assume that: $\delta(0, \sigma^*) = \frac{1}{2}$ and $\alpha_d(\omega_1, 0, \sigma^*) = \beta_d(\omega_1)$. Both F and V maintain their priors about θ when $s = 0$ is observed.

In the unique equilibrium both types of Mayor offer the subsidy, the Firm accepts it and makes its first-period investment in the local project, but the Mayor is not necessarily re-elected. That happens only if things turn out well for the voter in the first period, and if they do not, the election of a challenger as Mayor halts the project as the subsidy is withdrawn/reduced and the Firm reacts by abandoning it and going to its next-best option in period 2.

That the aforementioned equilibrium is unique also implies there are no equilibria that are fully revealing – that is, different types of Mayor taking different actions (such as $\sigma = (0, v)$ or $\sigma = (v, 0)$). Such signalling is not sustainable in equilibrium. If only the type l Mayor offers the subsidy then the voter, upon observing a subsidy offer, would prefer to elect the challenger than an incumbent Mayor now know to be a low type, since that also frees the voter from financing the subsidy. Thus, an l Mayor would not offer it in equilibrium, but pretend to be a high type by not offering it. Alternatively, if only the type h Mayor offers the subsidy then the voter, upon observing a subsidy offer, would happily elect the incumbent regardless of the realized state (i.e. ω_1). However, anticipating the voter's inference and subsequent actions, any l type Mayor would have an incentive to deviate by offering a subsidy (and pretend to be a h type) also. Less obviously, it is also not equilibrium behavior for both types of Mayor to not offer the subsidy in equilibrium. If either one were to deviate from such a strategy and offer it, the fact that both the voter and firm infer that both Mayor types have an incentive to do so implies that this will induce the firm to respond with an initial investment, which in turn increases the likelihood that $\omega_1 = 1$, which still gets the subsidy-offering Mayor re-elected. This is an opportunity that neither type of Mayor would pass up, so the belief that either type will deviate is correct, and this eliminates the possibility of a no-subsidy PBE.[8] Intuitively, when the realization of ω_1 is sufficiently informative and significant for the voter's payoff, the temptation to offer a subsidy that induces investment is too much for the Mayor to pass up, even though it does not guarantee re-election.

4.2 Pure strategy PBE for all q values

The type of PBE that can arise in the model more generally depends on the value of q, as we will now show.

Define a critical value of q, by:

$$q^o \equiv \frac{p_C^I - \Phi(p_{\theta I}, v, I)}{p_C^I - p_C^\phi}$$

where $\Phi(p_{\theta I}, v, I) = \beta_I(1)\, p_{hI} + [1 - \beta_I(1)]\, p_{lI} - v = \left(\frac{p_{hI}}{p_{hI}+p_{lI}}\right) p_{hI} + \left(\frac{p_{lI}}{p_{hI}+p_{lI}}\right) p_{lI} - v$.

This expression is in fact the expected payoff to the Voter of re-electing the incumbent Mayor after observing $(s, d, \omega_1) = (v, I, 1)$ in period 1.

Note that assumption C2 implies $\Phi(p_{\theta I}, v, I) > p_C^\phi$, and therefore that $q^o < 1$, but it also implies that q^o is negative if $p_C^I < \Phi(p_{\theta I}, v, I)$ - which is not ruled out by any of our assumptions - in which case $q > q^o$ necessarily. The importance of this critical value of q, the probability the Firm has a better option available than the (unsubsidized) local project, is apparent from the next result.

Proposition 2. *If conditions (??), and (??) hold, along with conditions C1,*

[8] This argument holds generally for any $q < 1$, while a variant of it utilizing the refinement of divinity (Banks and Sobel, 1987) holds when $q = 1$.

C2 and C3, then the following hold:

1. *There are no PBE in which the strategy of M is fully revealing for any value of q.*

2. *There is a PBE in pure strategies in which the Mayor chooses $\sigma = (0, 0)$ if and only if $q < q^o$. For this PBE, the full strategies are:*

   Mayor, $\sigma = (0, 0)$; Firm, $d_H(s) \equiv \phi$, $d_L(s) \equiv I$; and Voter, $e(\omega_1, v, \phi) = \omega_1, e(\omega_1, v, I) \equiv 0,$ and $e(\omega_1, 0, d) \equiv \omega_1$.

3. *There is a PBE in pure strategies in which the Mayor chooses $\sigma = (v, v)$ if and only if $q > q^o$. For this PBE, the full strategies are:*

   Mayor, $\sigma = (v, v)$; Firm, $d_H(0) = \phi$, $d_H(v) = I$, $d_L(s) \equiv I$; and Voter, $e(\omega_1, s, d) \equiv \omega_1$.

This is our central result, and it has a number of implications. First, note that it subsumes Proposition 1 when $q = 1 > q^o$, and so only this Proposition is proved in the Appendix.

The logic underlying part 3 of the result is much like that of Proposition 1, except that when in fact $q < 1$,the equilibrium behavior laid out may involve the Mayor offering a subsidy to a firm of type L for a project that the firm would have undertaken with no subsidy. If the Mayor is re-elected after that, as he will be if things go well, then the taxpayer will be paying for a needless subsidy, in that she would have gotten the stochastic increase in well-being without paying the tax cost. In addition, the Mayor who offers the subsidy may in fact still be turfed out of office if his first term does not go well, and in that case a challenger will be elected, the subsidy withdrawn or eliminated, and the project may or may not go forward without it. If in fact $q^o < 0$ then this is the only *possible* set of predicted outcomes, but if $0 < q < q^o$ then in equilibrium no subsidy is offered, because if one were, the Voter would replace the Mayor for certain, and so in equilibrium the project goes forward, unsubsidized, only if it is in fact profitable for the firm. A firm with a better alternative does not invest locally.

Note further that this is all still independent of whether (**??**) holds or not. This is because at the time at which the Voter must decide whether to re-elect the mayor, she has more information about his ability than is assumed in (**??**), which is relevant from the perspective of her priors about the Mayor and Firm. However, part 3 of the result also implies that when subsidies are offered, the term 'industrial blackmail' has a different - but still negative - connotation from that of over-bidding for a footloose firm's local project. When a subsidy is offered, there is a positive probability (except when $q = 1$, as in Prop 1) that the project would in fact have been undertaken with no subsidy at all. The voter and mayor will never know this if the mayor is indeed re-elected, since both types of firms accept the subsidy and the project goes on. Only if the subsidy is offered and the mayor is turfed out - which happens with positive probability - and still the project continues will the voter know she escaped from paying a needless subsidy.

5 Some empirical implications

The unique PBE of Proposition 2 are derived under a number of conditions. Each condition was essential to observing subsidies in equilibrium, so they provide essentially some boundary conditions on the phenomenon. Since industrial subsidies are not promised by all politicians or in all places, we consider what our results might imply for when they do occur.

The most obvious implication of the proposition is that subsidies are more likely to occur when there is a high probability that the firm has an alternative to the local investment that is superior to it, absent any subsidy, since the offer of a subsidy is an equilibrium outcome only if $q > q^o$. This suggests, in addition, that firms have an incentive to convey to policy-makers that they have such an alternative available. This line of thinking can be extended by noting that q^o, whose value depends only on model parameters, is increasing in both v and p_h. Thus, holding constant the (stochastic) impact of the local project on the citizenry, then the bigger the subsidy must be to induce investment by a type H firm, the less likely is that subsidy to be offered. In addition, holding all aspects of the project constant, the better is the challenger's expected ability in office, the less likely is the incumbent to offer the subsidy.

A second prediction is that subsidies are more likely to occur in electorally competitive jurisdictions. In an incumbent is certain to win re-election, then they have little incentive to strike deals with a firm in exchange for a electorally beneficial project. The same holds if the incumbent is expected to lose to a challenger, in which circumstance the firm may also be unwilling to commit resources to a project whose subsidy may be tied to a politician who is unlikely to be re-elected. Work by Carvalho (2014), Bertrand et al. (2018), and Bandiera-de-Mello (2018) show, in both France and Brazil, how investment and job creation by firms and incentivized by governments, were systematically targeted at districts that were more electorally contested.

A third prediction follows from inspection of the conditions C1 and C2. As noted, these both require that the answer to 'how did I do during the mayor's term?' is an important input into the voter's updated beliefs about their mayor, and therefore also their voting decision. This provides a different take on the Jensen and Malesky finding that subsidies are more likely in cities with 'strong mayors' than in those with city managers. C1 and C2 are less likely to hold in a city with a non-elected city manager, since the voter knows the state in period 1 is less dependent on the mayor's ability than in a city with a strong mayor. Specifically, it is easy to show that

$$\frac{d}{dp_{ld}}\left[\beta_d(1) - \beta_d(0)\right] < 0 < \frac{d}{dp_{hd}}\left[\beta_d(1) - \beta_d(0)\right]$$

and therefore a decrease in the value of $p_{hd} - p_{ld}$ results in a decrease in the size of the interval $\beta_d(1) - \beta_d(0)$, as well as a decrease in the size of the interval $[\beta_I(0)p_{hI} + [1 - \beta_I(0)]\, p_{lI}] - [\beta_I(1)p_{hI} + [1 - \beta_I(1)]\, p_{lI}]$. This implies first that C1 holds for a smaller set of p_h values, and second that C2 holds for a smaller set of $p_C^{\phi} + v$ values. In addition, p_C^{ϕ} is increasing in the value of $p_{h\phi} - p_{l\phi}$, so that taken together this implies that a decrease in the impact the a high-quality incumbent has on the voter's well-being (through its impact on ω_t) as represented by a decrease in $p_{hd} - p_{ld}$ (for either or both of $d = \phi, I$), decreases the likelihood of a subsidy offer.

Our result also suggests that subsidies are more likely to be offered in jurisdictions that are of sufficient population (or more exactly, sufficient tax base). First, note that jurisdictions that are too small in this sense will tend to lack the resources sufficient to induce a type H firm to invest.

This follows from consideration of condition C3 and C2. Note that the first provides a lower bound on how small a subsidy the type H firm is willing to accept to make the investment, since it can be re-written as:

$$v > (\pi_H - \pi_I) + \eta + \frac{\gamma}{\frac{1}{2}\,(p_{hI} + p_{lI})}$$

The subsidy must be large enough to compensate the firm for the opportunity cost of the project $(\pi_H - \pi_I)$, the continuing operating costs of the project η, as well as a portion of the sunk cost of investment $\frac{\gamma}{\frac{1}{2}(p_{hI} + p_{lI})}$.

Now consider the upper inequality of condition C2, but amend our model trivially so that the citizenry consists of n individual but identical voters, who must each - we will assume - pay $1/n$ of any subsidy granted, so that the upper inequality in C2 can be written as:

$$v/n < \beta_I(1)p_{hI} + [1 - \beta_I(1)]\, p_{lI} - p_C^{\phi}$$

A value of v large enough to satisfy C3, will thus be more likely to fail to satisfy this inequality in a smaller jurisdiction, ceteris paribus.

6 Conclusion

This paper developed a model of the political economy of subsidies for private investment that is centered on informational asymmetries between politicians and voters and firms and voters. We showed that subsidies arise in equilibrium but only under specified conditions, as a result of 'signal-jamming' by the politician. We also argued that the model has interesting empirical implications.

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Appendix

Although we state it separately, Proposition 1 actually follows from Proposition 2, so we prove only the latter here.

First, define the period 2 expected payoff to the voter as a function of his choice of e, and of the history she has seen, (s, d, ω_1) and of the strategy σ of the Mayor. Write this as

$$\begin{aligned} U^2\left(e, \omega_1, s, d|\sigma\right) &= \Pr\left\{\omega_2 = 1|e, \omega_1, s, d|\sigma\right\} \cdot 1 - \varphi(d)es \\ &\equiv \chi\left(e, \omega_1, s, d|\sigma\right) - \varphi(d)es \end{aligned}$$

And note the following:

$$\chi\left(0, \omega_1, s, d|\sigma\right) = \xi(s, d, e)\left[p_h p_{hI} + (1-p_h)\, p_{lI}\right] + \left[1 - \xi(s, d, e)\right]\left[p_h p_{h\phi} + (1-p_h)\, p_{l\phi}\right]$$

where $\xi(s, d, e) =$ the probability that the project continues in period 2, given s, d and the voter's choice of e.
and analogously,

$$\begin{aligned} \chi\left(1, \omega_1, s, d|\sigma\right) &= \xi(s, d, e)\left[\alpha\left(\omega_1, s, d|\sigma\right) p_{hI} + \left(1 - \alpha\left(\omega_1, s, d|\sigma\right)\right) p_{lI}\right] \\ &\quad + \left[1 - \xi(s, d, e)\right]\left[\alpha\left(\omega_1, s, d|\sigma\right) p_{h\phi} + \left(1 - \alpha\left(\omega_1, s, d|\sigma\right)\right) p_{l\phi}\right] \end{aligned}$$

Note further that $\xi(s, \phi, e) \equiv 0$ and $\xi(0, I, e) \equiv 1$: if there is no initial investment there can be no continuation of the project in period 2, and if there is investment in period 1 with no subsidy, it must be that $\alpha = l$ and so the project will continue whatever the voter does. Finally, $\xi(v, I, 1) = 1$. If a subsidy was offered and investment made, and the voter re-elects the incumbent, the project continues with certainty, as both types of firm will continue it.

Proof of i):

Suppose that $\sigma = (0, v)$ so that seeing $s = v$ reveals that $\theta = h$.

We know that $d_L \equiv I$ and $d_H(0) = \phi$ in any PBE. $d_H(v)$ may be either I or ϕ in such a PBE.

We first determine the strategies of V that must follow if this is to be a PBE.

If the voter sees $(s, d) = (0, \phi)$,

Then V knows that $\theta = l$, and $\tau = H$, and so $U^2(0, \omega_1, 0, \phi|\sigma) = p_C^\phi = p_h p_{h\phi} + (1 - p_h) p_{l\phi} > p_{l\phi} = U^2(1, \omega_1, 0, \phi|\sigma)$, so $e^*(\omega_1, 0, \phi|\sigma) \equiv 0$.

If the voter sees $(s, d) = (0, I)$

Then V knows that $\theta = l$ and $\tau = L$, so the project will continue, so $U^2(0, \omega_1, 0, I|\sigma) = p_h p_{hI} + (1 - p_h) p_{lI} = p_C^I > p_{lI} = U^2(1, \omega_1, 0, I|\sigma)$, and again $e(\omega_1, 0, I|\sigma) \equiv 0$

This means that M_l's payoff in such a PBE must be $U_l^* = 0$, no matter how F_H reacts to $s = v$.

We now show that M_h must get a positive expected payoff in such a PBE.

i) Suppose first that $d_H(v) = \phi$.

If the voter sees $(s, d) = (v, \phi)$,

Then V knows that $\theta = h$ and $\tau = H$, and so $U^2(0, \omega_1, v, \phi|\sigma) = p_C^\phi < p_{h\phi} = U^2(1, \omega_1, v, \phi|\sigma)$ so $e^*(\omega_1, v, \phi) \equiv 1$, and therefore M_h's PBE payoff is $U_h^* = 1$.

ii) Now suppose that $d_H(v) = I$

If the voter sees $(s, d) = (v, I)$, then she knows that $\theta = h$ and $U^2(0, \omega_1, v, I|\sigma) = \xi(v, I, 0)\left[p_C^I\right] + [1 - \xi(v, I, 0)]\, p_C^\phi$, and $U^2(1, \omega_1, v, I|\sigma) = p_{hI} - v$.

As $e = 0$ implies the project continues iff $\tau = L$, it follows that $\xi(v, I, 0) = 1 - q$,so $U^2(0, \omega_1, v, I|\sigma) = (1 - q)p_C^I + qp_C^\phi \equiv Q > 0$

This means that, independently of the ω_1 realization again,

$$e = 1 \Leftrightarrow (1 - q)p_C^I + qp_C^\phi < p_{hI} - v$$

This inequality can go either way under our assumptions so far. [C2 implies $p_{hI} - v > p_C^\phi$, but $p_{hI} - v$ could be less than $p_C^I = p_h p_{hI} + (1 - p_h) p_{lI}$.]

Suppose that $e^*(\omega_1, v, I) \equiv 0$ because $Q < p_{hI} - v$. This would mean that in fact $d_H(v) = \phi$ in PBE, and we showed above that in that case, $U_h^* = 1$ and this cannot be a PBE because then M_l would deviate to $s = v$.

Suppose instead that $e^*(\omega_1, v, I) \equiv 1$, as $Q > p_{hI} - v$. Then indeed it will be that $d_H(v) = I$ in a PBE, and this implies again that $U_h^* = 1$ so this cannot be a PBE again, for the same reason.

We now show there cannot be a PBE in which M uses $\sigma = (v, 0)$.

First we claim that in a PBE with this σ, it must be that $d_H(v) = \phi$ - the H firm type does not invest when offered a subsidy.

Spose not, b.w.o. c., so $d_H(v) = I$

Then when the Voter sees (v, I) his beliefs are that $\theta = l$ and his beliefs about τ remain at q, since both Firm types respond to v with $d = I$

If V then responds to this with $e = 0$ her payoff is $U(0, v, I|\sigma, d) = qp_C^\phi + (1 - q) p_C^I = Q$ as only the L firm will continue the project.

If, however, V responds with $e = 1$ her payoff is $U(1, v, I|\sigma, d) = p_{lI} - v$, as the project continues for certain but with a type l Mayor and the voter must pay for the subsidy. However, $p_C^\phi > p_{lI} - v$ by C2 and $p_C^I > p_{lI} > p_{lI} - v$ so that the Voter will choose $e = 0$ independently of the realization of ω_1. That, however, implies that the type h firm's strategy must be that $d_H(v) \equiv \phi$ if this is to be a PBE.

So, if the Voter sees $(s, d) = (v, I)$ she knows that $\theta = l$ and $\tau = L$ and so $U(0, v, I) = p_C^I > p_{lI} - v = U(1, v, I)$, and so chooses $e = 0$.

If the Voter sees $(s, d) = (v, \phi)$ she knows that $\theta = l$ and $\tau = H$ and so $U(0, v, \phi) = p_C^\phi > p_{l\phi} = U(1, v, \phi)$ and so chooses $e = 0$ again.

This implies the PBE payoff to M_l will be $U_l = 0$

When the voter sees $(s,d) = (0,I)$ she knows $\theta = h$ and $\tau = L$ and so $U(0,0,I) = p_C^I < p_{hI} = U(1,0,I)$ so $e = 1$

When the voter sees $(s,d) = (0,\phi)$, she knows $\theta = h$ and $\tau = H$ and so $U(0,0,\phi) = p_C^\phi < p_{h\phi} = U(1,0,\phi)$ so again $e = 1$ and this implies the payoff to M_h in such a PBE will be $U_h = 1$, and this implies it cannot be a PBE, as M_l will prefer to play $s = 0$.

This proves i).

Proof of ii)

Suppose there is a PBE in which M plays $\sigma = (0,0)$. Then it must be that the firm responses in equilibrium are $d_L(0) \equiv I$ and $d_H(0) = \phi$.

This in turn means that if the voter sees $(s,d) = (0,\phi)$ then she knows that $\tau = H$ and so $U^2(1,\omega_1,0,\phi) = \beta_\phi(\omega_1)p_{h\phi} + \left(1 - \beta_\phi(\omega_1)\right) p_{l\phi}$ and $U^2(0,\omega_1,0,\phi) = p_C^\phi$ and C1 then implies that $e(\omega_1,0,\phi) = \omega_1$.

If the Voter sees instead $(s,d) = (0,I)$ then it knows that $\tau = L$ and the project will continue, so the voter's possible payoffs are:

If $e = 1$, then $U^2(1,\omega_1,0,I) = \beta_I(\omega_1)p_{hI} + [1 - \beta_I(\omega_1)] p_{lI}$ and if $e = 0$ then $U^2(1,\omega_1,0,I) = p_C^I = p_h p_{hI} + (1 - p_h) p_{lI} > p_C^\phi$.

This implies $e = 1$ is optimal iff $\beta_I(\omega_1) > p_h$ and so our C1 implies that $e(\omega_1,0,I) = \omega_1$.

This in turn means that the PBE payoff for M_θ is

$$U_\theta^* = q p_{\theta\phi} + (1 - q) p_{\theta I}$$

The above arguments demonstrate that the Firm and voter PBE strategies in response to the Mayor's strategy are:

$$\begin{aligned} F_L(s) &\equiv I, F_H(0) = \phi \\ e(\omega_1,0,\phi) &= \omega_1 = e(\omega_1,0,I) \end{aligned}$$

. For this to be a PBE it is only necessary to show that neither Mayor type can increase their payoff by deviating to $s = v$. Assume as before that if such a deviation occurs, both the Firm and Voter assume that it is equally likely to be either M_θ; it will be shown below that either both Mayor types have an incentive to make this deviation, or both types do not.

Now define $Q(q) \equiv q p_C^\phi + (1-q)\, p_C^I$ and notice that since $p_C^\phi < p_C^I$, this is *decreasing in* q.

Q is in fact the expected payoff to the Voter if she chooses $e = 0$ after seeing $(s, d) = (v, I)$, if the voter maintains her prior beliefs (q) about τ. Since F_H will not invest in the project without the subsidy, the payoff to the Voter in period 2 will be p_C^ϕ, the likelihood the challenger is of type h given $x = 0$.

Further, $U^2(1, \omega_1, v, I) = \beta_I(\omega_1)\, p_{hI} + [1 - \beta_I(\omega_1)]\, p_{lI} - v$, which is the expected payoff to a Voter who has seen (v, I, ω_1) if she chooses $e = 1$, as the project will continue in period two independent of the firm type. Note further that C2 implies that $U^2(1, 0, v, I) = \beta_I(0)\, p_{hI} + [1 - \beta_I(0)]\, p_{lI} - v < p_c^\phi \leq Q(q)$ for any $q \in [0, 1]$.

However, $U^2(1, 1, v, I) = \beta_I(1)\, p_{hI} + [1 - \beta_I(1)]\, p_{lI} - v > p_C^\phi$ by C2, but this implies nothing about $U^2(1, 1, v, I)$ relative to $Q(q)$. One possibility is that $U^2(1, 1, v, I) > p_C^I$, in which case $U^2(1, 1, v, I) > Q(q)$ for all $q \in [0, 1]$.

However, since $Q(q)$ is decreasing in q and $Q(1) = p_C^\phi < U^2(1, 1, v, I)$, it follows that there is a value of $q < 1$, call it q^o, such that $U^2(1, 1, v, I) = Q(q^o)$ and

$$U^2(1, 1, v, I) < Q(q) \Leftrightarrow q < q^o$$

We thus define a critical value for q, which is

$$q^o = \frac{p_C^I - U^2(1, 1, v, I)}{p_C^I - p_C^\phi}$$

which is less than 1, as C2 implies $U^2(1, 1, v, I) > p_C^\phi$, and is negative (and irrelevant) if in fact $p_C^I < U^2(1, 1, v, I)$.

To determine whether or not the Mayor can deviate profitably to $s = v$, and so determine if this is a PBE, we consider two cases.

A. Assume $q^o > 0$ and consider values of $q < q^o$

Suppose now that either M_θ deviates to $s = v$.

First we ask whether it can be that F_H responds with $d_H(v) = I$.

Suppose so. Then upon seeing v, I the Voter is left with its priors about both the firm and the mayor, and so its payoffs are:

If the Voter chooses $e = 0$ then $U^2 = Q(q)$, but if she chooses $e = 1$ then $U^2 = U^2(1, \omega_1, v, I)$, and $q < q^o$ implies that $U^2(1, 0, v, I) < U^2(1, 1, v, I) < Q(q)$ and so $e \equiv 0$ is optimal. That in turn means that it cannot be sequentially rational for F_H to choose I in response to this deviation.

Given that the H Firm responds to a deviation to $s = v$ with $d_H(v) = \phi$, the payoffs to V in each possible node are as follows.

When $(s, d) = (v, I)$ the voter knows $\tau = L$ and so choosing $e = 0$ implies a voter payoff of $U^2 = p_C^I$ and $e = 1$ implies $U^2(1, \omega_1, v, I) = \beta_I(\omega_1)p_{hI} + [1 - \beta_I(w_1)]\, p_{lI} - v$

C.2 implies that $p_C^I > p_C^\phi > U^2(1, 0, v, I)$, so the best payoff the deviating mayor can get if the Firm is of type L is if $e(\omega_1, v, I) = \omega_1$, which is $p_{\theta I}$

If the voter sees $(s, d) = (v, \phi)$ then she knows $\tau = H$ and so choosing $e = 0$ implies $U^2 = p_c^\phi$ and $e = 1$ implies $U^2(1, \omega_1, v, \phi) = \beta_\phi(\omega_1)p_{h\phi} + \left[1 - \beta_\phi(\omega_1)\right] p_{l\phi}$ so that in fact $e(\omega_1, v, \phi) = \omega_1$ by C1.

This means that the *maximum* payoff M_θ can expect from this deviation is $qp_{\theta\phi} + (1 - q)p_{\theta I}$, which is equal to the PBE payoff, so this deviation is not profitable for either Mayor type, and we conclude that if $q < q^o$ then this is a PBE. Further, the reasoning above has shown that the PBE strategies of the Voter and Firm off the equilibrium path (assuming they maintain their priors) are as follows:

$$\begin{aligned} d_H(v) &= \phi \\ e(\omega_1, v, \phi) &= \omega_1, \text{and} \\ e(\omega_1, v, I) &= \omega_1 \text{ or } e(\omega_1, v, I) \equiv 0, \text{depending on whether} \\ \beta_I(1)p_{hI} + [1 - \beta_I(1)]\, p_{lI} - v &> \quad \text{or} \ < p_C^I \end{aligned}$$

B. Now assume $q > q^o$ and again we determine if either M_θ can deviate profitably to $s = v$.

Now we claim that F_H will respond to a deviation to $s = v$ with $d_H(v) = I$. To see this, note that if H chooses $d = \phi$ then his payoff is π_h for certain. Suppose instead he chooses I, as claimed.

Then the Voter, seeing v, I, is left with her priors about both θ and τ, and so her payoff from each action is as follows:

Choosing $e = 0$ implies $U^2 = p_C^I$ and choosing $e = 1$ implies $U^2 = \beta_I(\omega_1)\, p_{hI} + [1 - \beta_I(\omega_1)]\, p_{lI} - v = U^2(1, \omega_1, v, I)$ and since $q > q^o$, it follows that $e(\omega_1, v, I) = \omega_1$ and so F_H's payoff is greater than π_h, by our assumption C3.

This in turn means the payoff to M_θ from deviating is $p_{\theta I} > U_\theta^* = qp_{\theta\phi} + (1 - q)\, p_{\theta I}$, and we have shown that it is profitable for both Mayor types to deviate to $s = v$ when $q > q^o$, which proves that $\sigma = (0, 0)$ can be part of a PBE *only* if $q < q^o$, thus completing the proof of ii).

Proof of iii)

Suppose that $\sigma = (v, v)$ is part of a PBE.

There are two possible responses of F_H to $s = v$ to consider.

i) $d_H(v) = \phi$

Then when the Voter sees v, I she knows that $\tau = L$ and so the payoffs for the Voter are:

If she chooses $e = 0$ then $U^2 = p_C^I$ as only F_L will choose $x = 1$ and if she chooses $e = 1$ then $U^2(1, \omega_1, v, I) = \beta_I(\omega_1) p_{hI} + [1 - \beta_I(\omega_1)] p_{lI} - v$ because both types of F will choose $d = I$.

Since $p_C^I > Q(q)$ for all q and $Q(q) > U^2(0)$, it follows that $e(0, v, I) = 0$.

By definition of q^o, it also follows that if $q < q^o$ then $U^2(1, \omega_1, v, I) < Q(q) < p_C^I$ so $e(1, v, I) = 0$ also.

If the Voter sees v, ϕ she knows that $\tau = H$ and so her payoffs are:

From $e = 0$ she gets $U^2 = p_C^\phi$ and from $e = 1$ it is $U^2(\omega_1) = \beta_\phi(\omega_1) p_{h\phi} + \left[1 - \beta_\phi(\omega_1)\right] p_{l\phi}$ so that C1 implies that $e(\omega_1, v, \phi) = \omega_1$

ii) Suppose instead that $d_H(v) = I$

Then the Voter will only see v, I and will maintain her priors about θ and τ so her payoffs will be:

If she chooses $e = 0$ then $U^2 = qp_C^\phi + (1 - q) p_C^I = Q(q)$ and if $e = 1$ then $U^2(\omega_1) = \beta_I(\omega_1) p_{hI} + [1 - \beta_I(\omega_1)] p_{lI} - v$

From the definition of q^o then, we have that if $q > q^o$ then $e(\omega_1, v, I) = \omega_1$ and if $q < q^o$ then $e(\omega_1, v, I) \equiv 0$

This allows us to prove the following claim:

Claim F: In any PBE with $\sigma = (v, v)$ the strategy of F_H must be:

$$d_H(v) = \begin{cases} I, \text{ if } q > q^o \\ \phi, \text{ if } q < qo \end{cases}$$

Proof of Claim F: The payoff to F_H from $d_H = \phi$ is always the same, π_H.

Suppose then that $q > q^o$. If F_H chooses I, then the analysis above shows that $e(\omega_1, v, I) = \omega_1$ and so the payoff to F_H is greater than π_H by condition C.3, so $d_H(v) = I$ as claimed.

If instead $q < q^o$, then the analysis above implies that if F_H chooses I, then $e(\omega_1, v, I) \equiv 0$ and the payoff to F_H is $\pi_H - \gamma$, so in fact $d_H(v) = \phi$.

This proves Claim F. ■

We now prove iii) in two cases.

A. Assume $q > q^o$

Claim F implies that $d_\tau(v) = I$ for both firm types, and that means the voter sees only v, I in equilibrium.

This means the payoffs to each possible voter strategy are:
If $e = 0$ then $U^2(0, \omega_1, v, I) = qp_C^\phi + (1-q)\, p_C^I = Q(q)$

If $e = 1$ then $U_V(1, \omega_1, v, I) = \beta_I(w_1)p_{hI} + (1 - \beta_I(\omega_1))\, p_{lI} - v$, so that $q > q^o$ implies that $e\,(\omega_1, v, I) = \omega_1$

This in turn implies that the equilibrium payoff to M_θ is $p_{\theta I}$.

The only thing left to determine whether this is a PBE is whether M_θ can get a higher payoff by deviating to $s = 0$. We assume F and V regard the deviation as being equally likely to come from either θ and so maintain their 1/2 prior, and we know the firms will respond with $d_H(0) = \phi$ and $d_L(0) = I$, and that leads to the following Voter behavior

If the Voter sees $0, I$ she knows that $\tau = L$ and the project will continue, so that

Choosing $e = 0$ gives her $U^2 = p_C^I$ and choosing $e = 1$ results in $U^2(\omega_1) = \beta_\phi\,(\omega_1)\, p_{h\phi} + \left[1 - \beta_\phi\,(\omega_1)\right] p_{l\phi}$ and therefore $e(\omega_1, 0, I) = \omega_1$ by C.1.

If the Voter sees $0, \phi$ she knows $\tau = H$ so that

If she chooses $e = 0$ then $U^2 = p_C^\phi$ and if $e = 1$ then $U^2(\omega_1) = \beta_\phi\,(\omega_1)\, p_{h\phi} + \left[1 - \beta_\phi\,(\omega_1)\right] p_{l\phi}$ and so $e\,(\omega_1, 0, I) = \omega_1$ by C.1 again.

This in turn means the payoff to M_θ from deviating to $s = 0$ is $U_\theta(0) = qp_{\theta\phi} + (1-q)\, p_{\theta I} < p_{\theta I}$ and so this is not a profitable deviation. This proves that in fact the strategies laid out in iii) are a PBE when $q > q^o$.

B. $q < q^o$

Claim F now implies that $d_H(v) = \phi$ and $d_L(v) = I$ in any PBE, so we have the following for the Voter's reactions:.

If the voter sees v, ϕ she knows $\tau = H$ and the project will not happen, so $e = 0$ implies a payoff of $U^2 = p_C^{\phi}$ and if $e = 1$ her payoff is $U^2(\omega_1) = \beta_{\phi}(\omega_1) p_{h\phi} + \left[1 - \beta_{\phi}(\omega_1)\right] p_{l\phi}$ and the fact that $q < q^o$ means that $e(\omega_1, v, \phi) = \omega_1$.

If the voter sees v, I she knows $\tau = L$ and the project will happen no matter what she does, so the payoff to choosing $e = 0$ is $U^2 = p_C^I$ and the payoff to choosing $e = 1$ is $U^2(\omega_1) = \beta_I(\omega_1) p_{hI} + [1 - \beta_I(\omega_1)] p_{lI} - v$ and $q < q^o$ means that $p_C^I > Q(q) > \beta_I(1) p_{hI} + [1 - \beta_I(1)] p_{lI} - v > \beta_I(0) p_{hI} + [1 - \beta_I(0)] p_{lI} - v$ so that $e(\omega_1, v, I) \equiv 0$.

Taken together this implies the payoff to M_θ from this potential PBE is $U_\theta^* = q p_{\theta\phi}$.

For this to be a PBE it must be that neither M_θ can do better by deviating to $s = 0$.If either type does so we have that $d_H(0) = \phi$ and $d_L(0) = I$.

If the Voter sees $0, \phi$ she knows that $\tau = H$ and so her payoffs are p_C^{ϕ} from choosing $e = 0$ and $\beta_{\phi}(\omega_1) p_{h\phi} + \left[1 - \beta_{\phi}(\omega_1)\right] p_{l\phi}$ from choosing $e = 1$,so that C1 implies that $e(\omega_1, 0, \phi) = \omega_1$.

If the Voter sees $0, I$ she knows that $\tau = L$ and so her payoffs are p_C^I from choosing $e = 0$ and $\beta_I(\omega_1) p_{hI} + [1 - \beta_I(\omega_1)] p_{lI}$ if she chooses $e = 1$, and so C.1 now implies that $e(\omega_1, 0, I) = \omega_1$, also. This in turn implies that the payoff to M_θ from this deviation is $q p_{\theta\phi} + (1 - q) p_{\theta I} > U_\theta^*$ since $q < q^o < 1$, and so there can be no PBE with $\sigma = (v, v)$ when $q < q^o$, proving the rest of iii), and completing the proof of Proposition 2.

■